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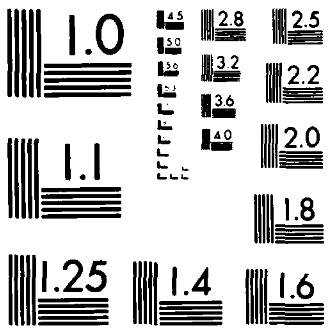

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The response of coastal sea level to local forcing by synoptic scale winds is usually assumed to be linear in wind stress. However, the response of sea level at St. Petersburg, Florida is found to be not linear, but to a power significantly less than one. The observed power-law exponent is 0.8 ± 0.1. The deviation from linearity in the power law is shown to be consistent with the effect of the quadratic form of the instantaneous bottom drag law. Therefore, the non-linear response should be true, to varying degrees, on many continental

20. shelves. Recognition of this non-linear sea level response to wind stress may allow significant improvement in the analysis of some sea-level problems.

On the wind stress - sea level power law

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June 1983

ABSTRACT

The response of coastal sea level to local forcing by synoptic scale winds is usually assumed to be linear in wind stress. However, the response of sea level at St. Petersburg, Florida is found to be not linear, but to a power significantly less than one. The observed power-law exponent is 0.8 ± 0.1.

The deviation from linearity in the power law is shown to be consistent with the effect of the quadratic form of the instantaneous bottom drag law. Therefore, the non-linear response should be true, to varying degrees, on many continental shelves. Recognition of this non-linear sea level response to wind stress may allow significant improvement in the analysis of some sea-level problems.

1. Introduction

In the course of analysing low frequency ($\lesssim 0.25$ cycles per day) adjusted sea level and wind velocity components on the West Florida Shelf, Cragg et al. (1983) (see also Cragg and Sturges, 1974) found that sea level varies as alongshore wind speed to the 1.3 ± .2 power. However, if sea level responds to alongshore wind stress, it is usually expected that the power law for wind speed should be close to 2. This result seems to be new. Recently, Sandstrom (1980), working on the Scotian shelf, noted that sea level was coherent at a higher significance level when wind speed alongshore was used rather than stress. However, he attributed this to "noise" introduced by the calculation of stress from the wind speed measurements.

It would be an important finding if sea level were not responding as a linear function of stress. The cross-shelf momentum balance for this time scale is believed to be nearly geostrophic (e.g., Huyer, et al., 1978; Allen and Kundu, 1978). On the West Florida Shelf the alongshore current is obviously forced by wind stress (Mitchum and Sturges, 1982). Therefore, we might expect sea level to also be forced by stress. In addition, an analytic solution of the nearshore flow (Csanady, 1978) suggests that sea level is linearly related to alongshore wind stress.

We use nearly 6 years of sea level data from St. Petersburg, Florida and meteorological data from nearby Tampa airport (Fig. 1) to examine the assumed relation

$$\eta = A\tau^n \qquad (1)$$

where η is sea level adjusted for the inverted barometer effect, and τ is the <u>alongshore</u> wind stress component. Both are evaluated at the coast. A and n are assumed constant. We restricted the tidal data to that obtained after

1972, when digital recording was introduced, as the noise level is substantially reduced.

2. Analysis.

The coordinate system is rotated 20°W of N to form a right-handed "alongshore, cross-shelf" system with x positive onshore and the origin at St. Petersburg. Wind stress is calculated using the usual quadratic law and a drag coefficient which varies linearly with wind speed; i.e., $C_D = (.8 + .065 W) \times 10^{-3}$ where W is wind speed in m/s (Wu, 1980). Sea level is adjusted for the static barometer effect. Cross-spectral analyses are done for sea level versus both wind stress components. Coherence with cross-shelf stress (not shown) is generally much smaller than with alongshore wind stress except in two narrow bands around .038 and .035 cycles per day (cpd). As expected, significant coherence is found with alongshore wind stress (Fig. 2). Two "bands" of coherence emerge: .012 to .025 cpd and .060 to .48 cpd. The main low in the coherence, near .04 cpd, lies in a region of very low energy in the wind forcing. This low coherence region, therefore, sems to be a feature only of the forcing and not of the dynamics of the shelf response. Both bands show an approximately .5 to 1 day lag of sea level relative to alongshore stress. The sea level data were consequently lagged by 15 hours, the average lag, relative to wind stress. Two band-pass filters were applied to isolate these two frequency domains (Table 1).

Numerical Experiments Using Artificial Data

A series of simple numerical experiments was run to determine a method of averaging the data to obtain an optimum determination of the power-law coefficient. The averaging is useful, because it suppresses random

variability from one event to the next. A log-log fit of sea level vs. alongshore wind stress is used to determine the exponent, "n", in (1). A non-linear least squares technique (Hartley, 1961) was also used for comparison, and yielded identical results. The log-log method requires that sea level and wind stress be of the same sign. For small data values, the "noise" can cause them occasionally to be of opposite sign. Neglecting such points, however, biases the noise; i.e., the least squares fit, if this effect is ignored, does not average to the best fit which would result if the noise were absent.

Artificial series of wind stress and sea level were generated, using a "known" power law, in the following manner. Alongshore wind stress was assumed to have a triangular variance-preserving spectrum (a close approximation to a highly smoothed spectrum) that peaks at .16 cpd and falls linearly to zero at .05 and .5 cpd. A random phase was used (following the idea of Thompson, 1973). The back transform to the time domain resulted in a reasonable alongshore wind stress series. This series was raised to the .5 power and multiplied by an amplitude factor (which happens to be 75) to produce sea level series similar to a measured one. Finally, random noise, with adjustable amplitude, was added to each series.

The series thus generated can then be used to test several methods of determining the exponent "n". Several results emerge. First, simply neglecting points where sea level and stress are of opposite sign does indeed bias the fit badly, as expected. Second, the noise level in the series used as the ordinate is much less important than that in the abscissa. This is expected, because the least squares fit used assumes all the error is in the "y" coordinate, and requires that the series with the lowest noise level be

used as the abscissa. Thus, because we consider the sea level to be the more accurately known data, we inverted the fitting method and determined the exponent from the coefficients of a log (stress) as a function of log (sea level) fit. Third, "batch averaging" the data improved the determination greatly. The best averaging scheme we found is one similar to that used by Scott and Csanady (1976) in determining a bottom friction parameter off Long Island. The actual "bins" used are shown in Table 2. Finally, it was found that separating the data into sub-series, averaging these, determining n for each subset, and then averaging these as independent determinations of n also permits significant improvement. Each year of the band-passed data was used as a subset; each yearly average of the exponent is weighted by the inverse square of its standard deviation. We were able to obtain an accurate fit to the artificial data with peak signal-to-noise ratio (SNR) lower than 5. We do not expect the data to have a SNR this low. However, the noise in any real series may not be uniform, or random, as was assumed in our experiments.

Determination of Power Law Coefficient.

In the higher frequency band-pass shown in Table 1 the data were averaged (Table 2) and n determined as discussed above. We find that $n = 0.8 \pm 0.1$.

There is the possibility that the basic response is a function of the stratification in the water column, which will vary from strongly stratified, in late summer, to well mixed in winter. To test for seasonal variability, the data were separated into two-month groupings (i.e., all January and February data, all February and March data, etc.) and n determined for each two-month period (Fig. 3). The December through March mean appears higher than the July-September mean. Unfortunately, the standard deviations are greater in summer, when the signal is smaller. Although the winter-summer

apparent difference is only at the level of one standard deviation, the result is suggestive.

The lower frequency band-pass had a smaller signal -about 5 cm sea level amplitude -as compared with approximately 30 cm in the higher frequency band. The averaging bins were correspondingly altered (Table 2). The power-law coefficient was found to be 0.8 ± 0.2. Examination for a seasonal trend seemed inappropriate due to the large sampling interval (Table 1) and the correspondingly fewer number of points in the two month subsets.

3. Discussion

The power-law result here is consistent with the results of Cragg, et al. (1974; 1983). We retain the conclusion that the exponent is significntly less than unity. This determination is based on more data, and of better quality. The two band-passes yield nearly the same n. Moreover, the phase lag of sea level to stress is very similar in the two bands as is the amplitude response function (not shown). The response in the "lower" frequency band is so rapid as to be clearly barotropic. It appears that the two bands are dynamically similar, and obviously related to the synoptic scale meterological forcing. In the following discussion we do not differentiate between the two frequency bands.

In order to understand a non-linear sea level response to wind forcing, it is natural to look for a term in the momentum equations that is non-linear and sufficiently large to cause observable effects. The non-linear terms in $\underset{\sim}{u}\cdot\nabla\underset{\sim}{u}$ are seen, by simple scale analysis, to be small and will be neglected. The alongshore component of the instantaneous bottom stress term, which is the leading candidate remaining, can be written

$$\tau_b^y = C_D \left| \underset{\sim}{u} \right| v \tag{2}$$

where $|\underset{\sim}{u}|$ is the current magnitude, v is the alongshore velocity component and C_D is a constant drag coefficient. This (non-linear) bottom stress term is known to be significant in the momentum balance on the West Florida Shelf (Mitchum and Sturges, 1982). In the nearshore region, the approximate balance in the synoptic band is between low frequency wind and bottom stresses, consistent with the theory of Csanady (1978). Thus, we write

$$\tau \approx C_D \left\langle |\underset{\sim}{u}| v \right\rangle \tag{3}$$

where τ is the low frequency alongshore wind stress and $\left\langle |\underset{\sim}{u}| v \right\rangle$ is the low frequency part of $|\underset{\sim}{u}|v$.

It is instructive to examine two limits of (3) to demonstrate how the bottom stress term in (3) may explain the observed power law. The first limit is the situation in which the high frequency currents (here, tides) are much more energetic than the low frequency ones. Csanady (1976) showed that in this case

$$\tau_b^y \approx rv \tag{4}$$

where r is a constant friction parameter equal to the drag coefficient times a high frequency root mean square current. Observations (e.g. Mitchum and Sturges, 1982) and numerical models (e.g. Hsueh, et al., 1982) show that to good approximation

$$v = \gamma\eta \tag{5}$$

where v is the low frequency, alongshore current, η is the low frequency coastal sea level fluctuation and γ is a constant of order one. Equations (4) and (5) thus imply that sea level and wind stress should be linearly related.

In the other limiting case, the high frequency currents are much less energetic than the low frequency currents. In this case, (2) shows that bottom stress varies approximately as alongshore current squared. Again, use of (5) implies that sea level should vary as the square root of wind stress in this limit.

These limits show that, depending upon the relative magnitudes of the high and low frequency currents, the power law coefficient should be between 0.5 and 1. The observed value of .8 ± .1 is nicely consistent with the fact that on the West Florida Shelf the high frequency (tidal) and low frequency (synoptic) currents are comparable in magnitude with the high frequency components being slightly larger (Mitchum and Sturges, 1982).

An attempt was made to quantify the hypothesis that the observed power-law coefficient, 0.8, is due to non-linearity in the bottom drag law. This was done by finding an approximate expansion for $|\underset{\sim}{u}|$ in the case where the tidal and synoptic currents are comparable in magnitude and using the observed currents, together with the observed variability in sea level, to compute a value for the power-law exponent. However, it proved to be impossible to obtain sufficient accuracy to significantly strengthen the result of the above paragraph. Despite the large uncertainty, the calculation did agree with the observed n and thus supports the hypothesis.

4. Conclusion

An analysis of coastal sea level and wind stress data from the West Florida Shelf at St. Petersburg shows that sea level does not vary linearly with alongshore wind stress as simple considerations would suggest. The observed non-linearity is found to be consistent with the fact that the bottom drag law is linear in (low frequency) velocity only in the limiting case for

which low frequency motions are much less energetic than high frequency ones. On the West Florida Shelf, where these components are comparable, the limiting case is inappropriate. Although the data used here are restricted to the West Florida Shelf, the non-linearity should be observed on other shelves. In other areas low frequency (synoptic scale) motions are often comparable to the tidal motions which primarily constitute the high frequency variability.

The finding that sea level does not respond linearly to alongshore wind stress can have important consequences for some types of data analysis. An example is any attempt to adjust a sea level series for synoptic wind forcing in order to examine variability which is not wind-forced.

Acknowledgements

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Figure Captions

Figure 1: Local bathymetry. Tide gauge is at St. Petersburg and meteorological data are from Tampa airport.

Figure 2: Spectra of adjusted sea level and alongshore wind stress. Calculated from 1096 daily values which have been tapered 10% on each end with a split cosine bell data window. The spectral estimates of sea level are shown by the dashed line (scale at right) while those of wind stress are shown by the solid line. The spectral smoothing window is 7 bands wide. Phase is shown only where the coherence squared is significant at the 95% level. Phase is positive when alongshore wind stress leads sea level.

Figure 3: Variability in the power law coefficient as a function of season. The error bars represent ± 1 standard deviation.

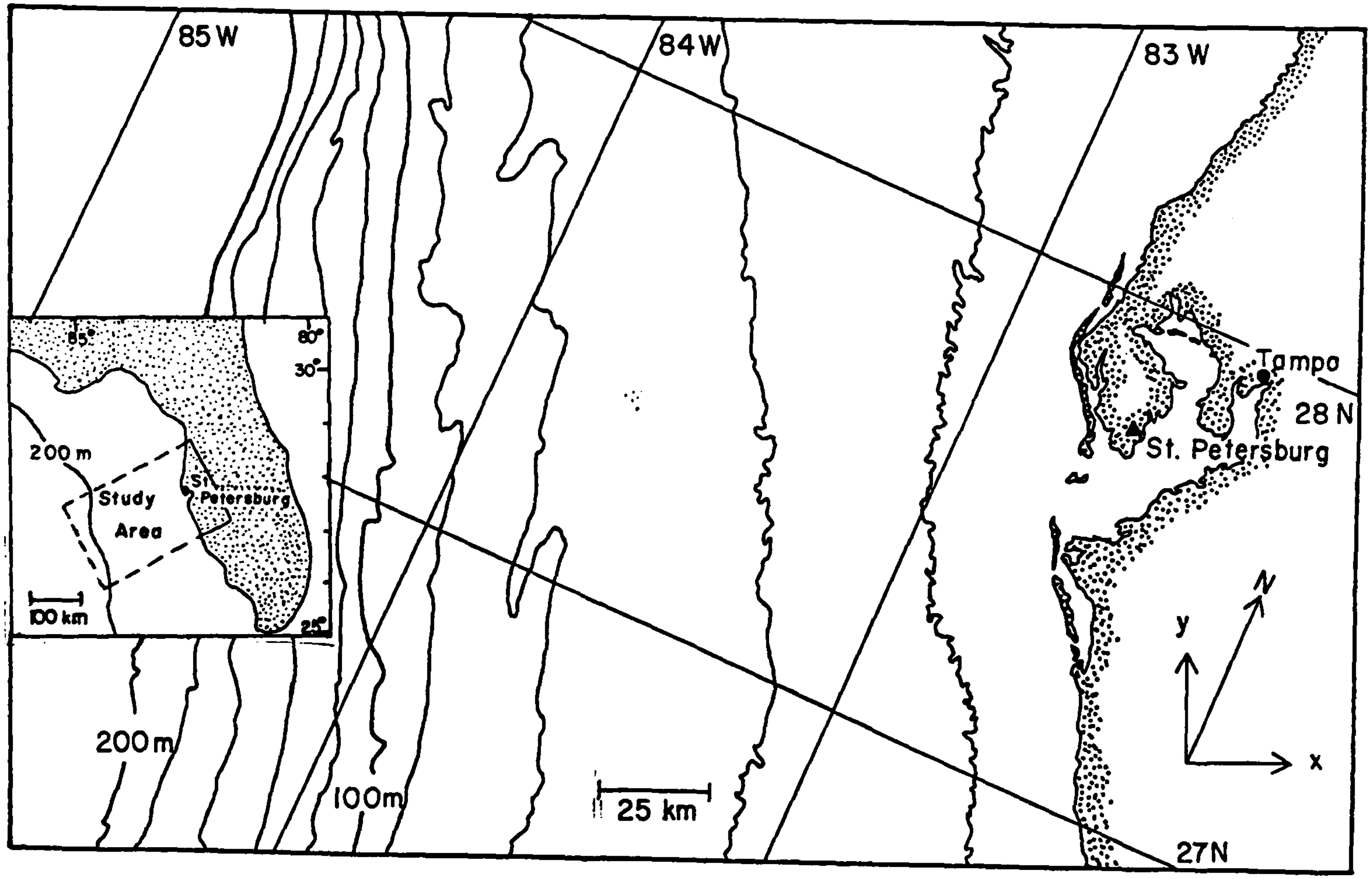

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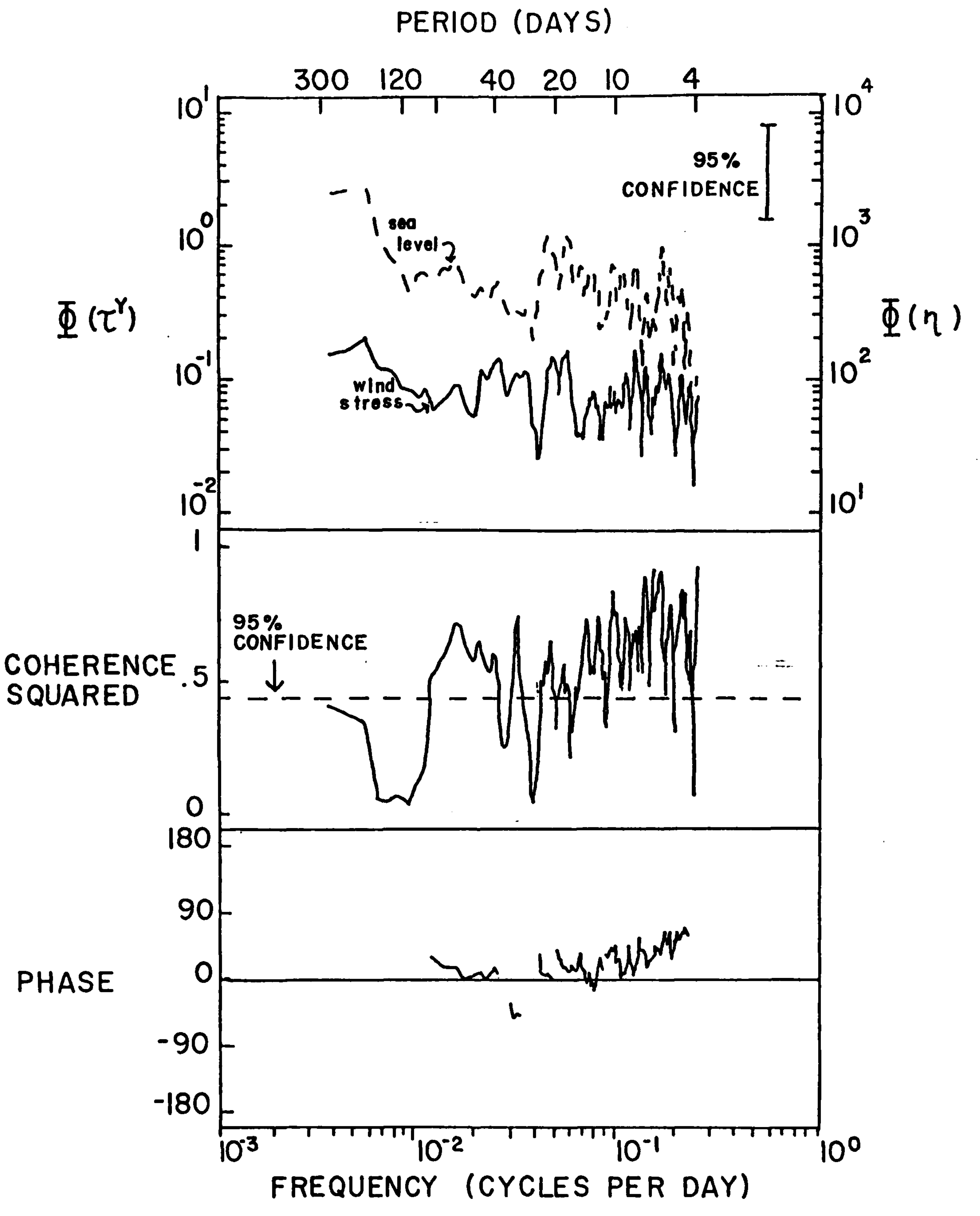
Fig. 1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CONFIDENCE
COHERENCE
SQUARED
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Fig. 2

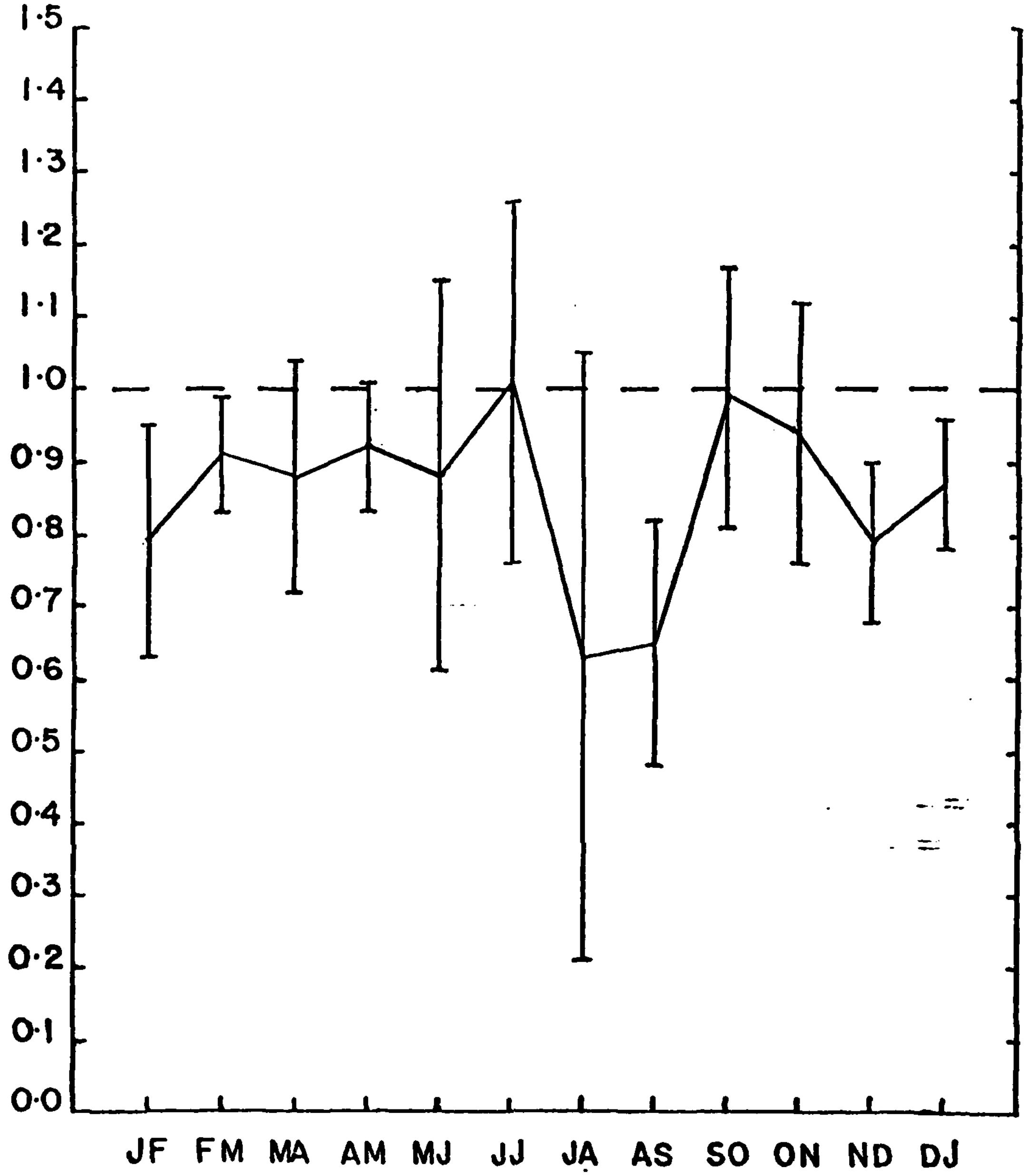
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Fig. 3

Table 1.

Details of the two band pass filters used. The lower frequency band was sampled weekly and had 220 points. The higher frequency band was sampled daily and had 2047 points.

Amplitude Response Function	Frequency (cpd) (Lower band)	Frequency (cpd) (Higher band)
$\leq$ 5%	.011	.045
50%	.014	.055
$\geq$ 95%	.016	.067
$\geq$ 95%	.022	.200
50%	.025	.333
$\leq$ 5%	.028	.450

Table 2

Batch averaging scheme for the band passes shown in Table 1. CWS, ASL are cross-shelf wind stress and adjusted sea level, respectively. The two numbers in any column represent lower and upper limits for that variable for the various bins. The lower limit is inclusive.

	Lower Freq. Band		Higher Freq. Band	
	CWS	ASL	CWS	ASL
Bin	(dynes cm^{-2})	(cm)	(dynes cm^{-2})	(cm)
1	-1,1	0,5	-1,1	0,1.5
2	-1,1	5,15	-1,1	1.5,3
3	-1,1	15,50	-1,1	3,6
4	-1,1	- 5,0	-1,1	-1.5,0
5	-1,1	-15,-5	-1,1	-3,-15
6	-1,1	-50,-15	-1,1	-6,-3
7	0,1	0,10	0,1	0,2.5
8	0,1	10,50	0,1	-2.5,6
9	0,1	0,50	0,1	0,6
10	0,1	-10,0	0,1	-2.5,0
11	0,1	-50,-10	0,1	-6,-2.5
12	0,1	-50,0	0,1	-6,0
13	-1,0	0,10	-1,0	0,2.5
14	-1,0	10,50	-1,0	2.5,6
15	-1,0	0,50	-1,0	0,6
16	-1,0	-10,0	-1,0	-2.5,0
17	-1,0	-50,-10	-1,0	-6,-2.5
18	-1,0	-50,0	-1,0	-6,0

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